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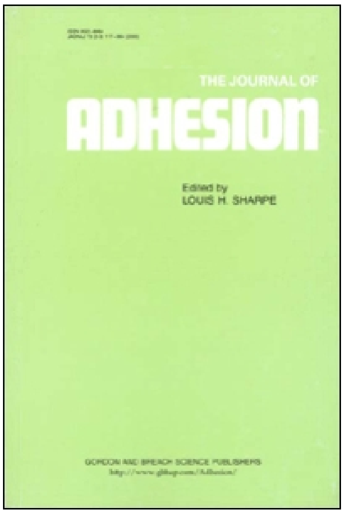

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Adherence of an Axisymmetric Flat Punch on a Thin Flexible Membrane

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An equilibrium theory of adhesion between an axisymmetric flat punch and a thin flexible membrane fixed at its circumference is derived. When the pulling force on the punch exceeds a "pull-off" threshold, a spontaneous delamination occurs at a finite radius leading to a complete separation between the adherends. Experiments on a simple materials system supported the analysis.

Keywords: Adhesion; Thin film; Punch; Delamination; Mechanics

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1. INTRODUCTION

Adhesion plays an important role in many important areas such as colloid coagulation, tribology, and biological systems. When two surfaces are drawn into contact by various physical and chemical surface forces, an adhesive contact is formed and elastic/plastic deformation occurs at the interface. To separate the two adherends, an external pulling force is usually required. It is the magnitude of this "pull-off" force and the mechanism of adhesion/decohesion which lay the foundation of adhesion theory. There has been much interest in the engineering community to measure adhesion between solid bodies in the past decades. One classical experiment of adhesion measurement

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was due to Kendall [1]. A circular flat punch was pulled away from an adhesive contact with an elastic continuum substrate. By considering a balance of elastic, potential and surface energies, it was shown that when an external pulling force on the punch exceeded a certain critical "pull-off" value, delamination occurred spontaneously until the punch was fully separated from the substrate. Maugis and Barquins discussed other punch geometries by the same argument [2]. The flat punch-continuum configuration appeared at about the same time as the JKR theory [3, 4]. This adhesion theory has proven to be very successful in accounting for adhesive contact between elastic bodies.

One limitation of the model, however, is due to the large rigidity required of the adherend. When a thin membrane is involved, *e.g.*, contact between two biological cells [5–6], it is necessary to modify the theory accordingly to account for the large flexibility and mechanical compliance of the film involved. In this paper, we consider the adherence between a flat rigid punch and a thin flexible membrane being constrained around a circular edge of fixed dimension. The membrane is assumed to be able to withstand stretching only, with neither bending nor shearing. A simple model based on both energy balance and linear elasticity will be derived to construct the contact mechanics. The new model, though distinct in many aspects, bears some resemblance with Kendall's model, especially the "pull-off" phenomenon. We will illustrate the new theory by simple experiments.

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2. THEORY

Figure 1 shows a thin, flexible and isotropic membrane with an elastic modulus, E, Poisson's ratio, ν, thickness, h, adhered onto a rigid plate with a central circular hole of radius, a. A cylindrical flat punch with the same radius, a, is brought to adhesive contact with the exposed diaphragm *via* the bore. An external force, F, is used to pull the punch from the membrane, while the contact area of radius, c, diminishes until complete separation at the punch-membrane interface. The plate-membrane adhesion is assumed to be strong and is not affected by any loading in the central diaphragm. For simplicity, viscoelastic behavior of the polymeric membrane is not considered in this paper. There are two existing methods to compute the constitutive relations: (i) by assuming an average stress on the membrane, while the membrane

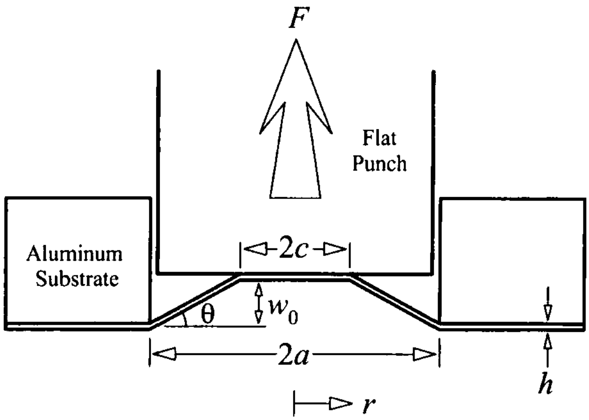

FIGURE 1 Adherence of an axisymmetric flat punch on a thin membrane. In an inverted V-peel geometry, a and c are replaced by l and l_c, respectively.

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geometry is deduced [7–8], and (ii) by assuming a straight edge conic geometry of the membrane, while the membrane stress is deduced [9–10]. These two approaches correspond to the upper and lower bounds of the delamination process.

2.1. Average Stress Approximation

In an average stress approximation, the radial and tangential stresses are assumed to be equal, or, $N_r = N_t = N$. The membrane profile $w(r)$ is derived by equating the vertical forces:

$$F = 2\pi r N \sin\theta \approx -2\pi r N \frac{dw}{dr} \tag{1}$$

which, after simple integration, yields

$$w = \frac{F}{2\pi N} \log\left(\frac{a}{r}\right) \tag{2}$$

The distance traveled by the punch is, therefore, $w_0 = (F/2\pi N)\log(1/\zeta)$ with $\zeta = c/a$. The membrane stress was shown earlier to be [7]:

$$\begin{aligned} N &= \frac{E'h}{2(a^2-c^2)} \int_c^a \left(\frac{dw}{dr}\right)^2 r\,dr \\ &= \left[\frac{E'h \log(1/\zeta)}{8\pi^2 a^2 (1-\zeta^2)}\right]^{1/3} F^{2/3} \end{aligned} \tag{3}$$

where $E' = E/(1-\nu^2)$. Substituting (3) into (2),

$$F = \frac{1}{(1-\zeta^2)\log^2(\zeta^2)}\left(\frac{4\pi E' h w_0^3}{a^2}\right) \tag{4}$$

Equation (4) represents a cubic constitutive relation *without* delamination along the punch-membrane interface, *i.e.*, constant ζ. Here the punch is pulling the membrane upwards, but since the adhesion is good, no delmaination occurs. In case of a point contact ($\zeta \approx 0$) without delamination, (4) reduces to the shaft-loaded blister solution [7].

For delamination to occur, the mechanical energy release rate of delamination, G_a, along the circular contact interface, is found to be

$$\begin{aligned} G_a &= -\left(\frac{1}{2\pi c}\right)\frac{d}{dc}\int w_0 dp \\ &= \frac{2-2\zeta^2-\zeta^2\log\zeta^2}{4\zeta^2(1-\zeta^2)\log(1/\zeta^2)}\left(\frac{Fw_0}{\pi a^2}\right) \end{aligned} \tag{5a}$$

which is identical to Williams' solution [8]. Alternative expressions can be obtained by substituting (4) into (5a) such that

$$G_a = \frac{2-2\zeta^2-\zeta^2\log\zeta^2}{4\zeta^2(1-\zeta^2)^{2/3}[\log(1/\zeta^2)]^{1/3}}\left[E'h\left(\frac{F}{4\pi E' h a}\right)^{4/3}\right] \tag{5b}$$

$$G_a = \frac{2-2\zeta^2-\zeta^2\log\zeta^2}{\zeta^2(1-\zeta^2)^2[\log(1/\zeta^2)]^3}\left[E'h\left(\frac{w_0}{a}\right)^4\right] \tag{5c}$$

At quasi-static equilibrium, $G_a = W$, the adhesion energy of the punch-membrane interface, which is a materials parameter. But when G_a exceeds W, delamination occurs along the interface and ζ decreases accordingly. The constitutive relation *with* delamination, $F(w_0)$, is governed by (5a) to (5c), and is shown in Figure 2 as a parametric curve QRST with a varying ζ. Points Q, R, S and T correspond to $\zeta = 1.00, 0.76, 0.19$ and 0.05, respectively. In a fixed load configuration where F increases gradually, no delamination is expected until the applied force reaches a critical force, F_{max}, at point Q. Since point Q represents the highest force in the equilibrium curve QRST, any further increase in F leads to a spontaneous crack growth followed by

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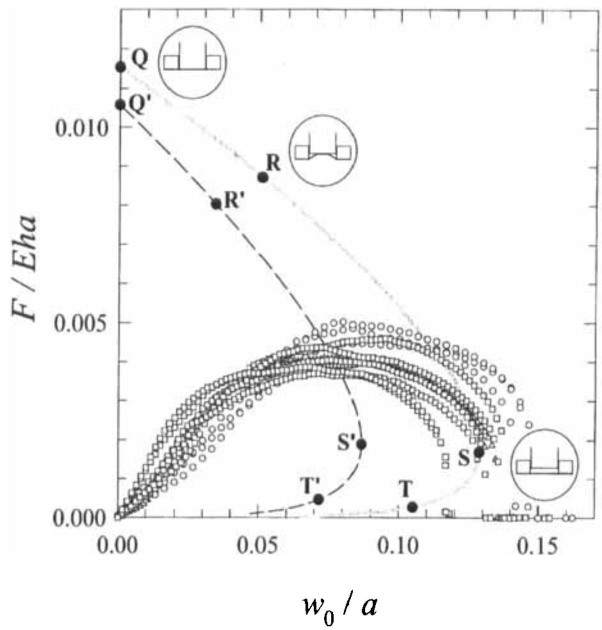

FIGURE 2 Elastic response (F/a) as a function of (w_0/a) for hole diameters 10 mm (circles), 20 mm (squares) and 25 mm (triangles). Cross-head speed was 1 mm · min^{-1}. The theoretical curves are shown for the average stress approximation (solid gray line) and the conical profile model (dashed line) with $W = 45\,\mathrm{J \cdot m^{-2}}$. At point S, spontaneous delamination occurs leading to complete separation at the punch–membrane interface.

a complete separation of the punch from the membrane. It can be shown that $F_{\max} = 5.6443\ [a(Eh)^{1/4}W^{3/4}]$ from (5b). The degree of stability can be improved by adopting a fixed grip configuration where w_0 is made to increase steadily. As the punch moves away from the membrane from Q to S, both F and c diminish, and the delamination is stable. Point S denotes the last point along the stable equilibrium branch, beyond which ($w_0 > w_0{}^*$) the delamination becomes unstable and grows spontaneously towards the center of the contact circle. At point S, where $(dw_0/dF) = 0$, it can be shown from (5a) to (5c) that

$$c^* = 0.1945\, a$$

$$F^* = 0.8508[a(Eh)^{1/4}W^{3/4}]$$

$$w_0^* = 0.8598[a(W/Eh)^{1/4}]$$

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assuming $\nu = 0.3$. The instability is known as the "pull-off" event. Branch ST is a solution to (5), but it represents an unstable equilibrium that is non-physical.

2.2. Conical Profile Approximation

An alternative method to derive the constitutive relation is the conical profile approximation. It was first applied to account for central point load acting on a blistering film [9], and was later modified for a spherical capped shaft replacing the point load [10]. In this paper, the theory is further modified for the punch-membrane system. A conical profile $w(r)$ with a straight edge is assumed in the non-contact region, *i.e.*, the membrane inclines at a fixed angle, θ, to the substrate. Thus,

$$\begin{aligned} w(r) &= (a-r)\tan\theta \quad \text{for } r > c \\ &= (a-c)\tan\theta = w_0 \quad \text{for } r \le c \end{aligned} \tag{6}$$

The law of energy conservation shows that

$$\int_c^a \left[\frac{F}{2\pi r} + N_r \frac{dw}{dr}\right]\left(\frac{dw}{dr}\right) 2\pi r dr = 0 \tag{7}$$

Substituting (6) into (7), and by the principle of virtual work, the radial stress, N_r, in the membrane was found to be:

$$\begin{aligned} N_r = &-\frac{Eh\tan^2\theta}{4}\log r \\ &+\frac{1}{r^2}\left[\frac{P}{2\pi(a+c)\sin\theta} - \frac{Eh\tan^2\theta}{4(a^2-c^2)}a^2c^2\log\frac{a}{c}\right] \\ &+\left[\frac{P}{2\pi(a+c)\sin\theta} - \frac{Eh\tan^2\theta}{4(a^2-c^2)}(a^2\log a - c^2\log c)\right] \end{aligned} \tag{8}$$

Note that N_r is undefined when c vanishes, though it will become apparent later that an instability in delamination sets in before that happens. Satisfying the boundary conditions, $2\pi a N_r|_{r=a}\sin\theta = 2\pi c N_r|_{r=c}\sin\theta = F$, and assuming small debonding angle with $\theta \approx 0$, it can be shown by (8) that

$$F = \frac{1-\zeta^2-\zeta\log\zeta^2}{(1-\zeta)^4}\left(\frac{\pi E h w_0^3}{4a^2}\right) \tag{9}$$

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Both average stress and conical profile approximations result in a cubic constitutive relation *without* delamination (*cf.* (4) and (9)). For delamination to occur, the mechanical energy release rate, G_c, can be deduced using the same method as in the previous section:

$$G_c = \frac{1 - \zeta^2 - \log\zeta - 3\zeta\log\zeta}{\zeta(1-\zeta)(1-\zeta^2-\zeta\log\zeta^2)}\left(\frac{Fw_0}{\pi a^2}\right) \tag{10a}$$

$$= \frac{(1-\zeta^2)^{1/3}(1-\zeta^2 - \log\zeta - 3\zeta\log\zeta)}{\zeta(1-\zeta^2-\zeta\log\zeta^2)^{4/3}}\left[Eh\left(\frac{4F}{\pi Eha}\right)^{4/3}\right] \tag{10b}$$

$$= \frac{1-\zeta^2 - \log\zeta - 3\zeta\log\zeta}{\zeta(1-\zeta)^5}\left[\frac{Eh}{4}\left(\frac{w_0}{a}\right)^4\right] \tag{10c}$$

using (9). At quasi-static equilibrium, $G_c = W$. The constitutive relation *with* delamination $F(w_0)$ is shown as a dashed curve Q′ R′ S′ T′ in Figure 2. Points Q′, R′, S′ and T′ correspond to $\zeta = 1.00$, 0.76, 0.19 and 0.05, respectively. A "pull-off" event is also predicted here. From (10a) to (10c), it can be shown that

$$F_{\max} = 2.318[a(Eh)^{1/4}W^{3/4}]$$

$$c^* = 0.1932a$$

$$F^* = 0.4211[a(Eh)^{1/4}W^{3/4}]$$

$$w_0^* = 0.5219[a(W/Eh)^{1/4}]$$

The contact mechanics is basically similar to that under the average stress approximation in terms of both fixed load and fixed grip configurations. The fact that these quantities derived here are different is due to the contrasting mechanical compliance. When the non-contact annulus is of negligible width ($c \approx a$), $G_a \approx G_c \approx (3/4)$ $(Fw_0/\pi a^2)$ (*cf.* (5a) and (10a)).

It is noted that the above theories are only valid when the delamination drives into the interface causing θ to decrease towards zero. When θ is large and $\zeta \approx 1$, the model will not be applicable in

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the initial delamination. The details are, however, beyond the scope of this paper.

3. EXPERIMENT AND RESULTS

A model interface was fabricated by adhering a commercially-available sticky tape onto a polished aluminum plate. A pressure sensitive adhesive tape was chosen for convenience in sample preparation. The aluminum plate was 2 mm thick with a pre-drilled hole of diameter $2a = 10$, 20 and 25 mm (Fig. 1). The elastic modulus and thickness of the membrane were measured to be 1.1 GPa and 60 μm, respectively. Aluminum cylinders with a polished flat end of diameter slightly smaller than the hole dimension at 9.70, 19.48 and 24.52 mm were put in contact with the exposed tape, while the other end was attached to the load cell of a universal testing machine. The cylinder was pulled away from the substrate at a cross-head speed of $1\,\mathrm{mm} \cdot \mathrm{min}^{-1}$. The external force was measured as a function of separation between the cylinder and the substrate.

Figure 2 shows the measurement of (F/Eha) as a function of (w_0/a) for various hole sizes. All data fell in the same region regardless of the hole dimension. Upon initial loading, the load increasel monotonically until a maximum was reached. There was some minor delamination at this stage due to the high debonding angle. The contact circle remained roughly the same as the punch surface. As the punch displacement increased further, the mechanical energy release rate eventually exceeded the adhesion energy and a delamination drove into the punch-membrane interface. As the contact circle shrank further, the applied load diminished until instability set in at the critical "pull-off" point, S. Here the punch displacement reached its maximum and the force dropped rapidly to zero. A complete separation of the punch from the membrane was observed with a small but finite contact radius. The adhesion energy of the dissimilar interface was measured earlier to be in the range between $W = 35$ and $50\,\mathrm{J} \cdot \mathrm{m}^{-2}$ using a standard 90° peel test and shaft-loaded blister test [9]. Putting $W = 45\,\mathrm{J} \cdot \mathrm{m}^{-2}$. The theoretical curves for the average stress approximation (solid line) and the average stress model (dashed line) are shown in Figure 2. Post-mortem observation

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showed no plastic yielding of the membrane after delamination. The loading process did not affect the surrounding tape bonded directly to the plate.

4. DISCUSSION

The existence of a predicted "pull-off" event in the punch-membrane configuration is remarkable, alluding to the similar feature in the punch-continuum system [1]. When a flat punch in an adhesive contact with a semi-infinite elastic solid (subscript *es*) or a rigid coating (subscript *rc*) is pulled away from the substrate by a force, F, the substrate material deforms linearly as

$$F = f(E, h, \nu, a) w_0$$

where $f_{es} = 2Ea/(1-\nu^2)$ and $f_{rc} = \pi a^2 \kappa / h$ are the two force constants, E is the elastic modulus of the continuum and κ the bulk modulus of the coating. A "pull-off" event is expected when the applied force reaches

$$F_{\max} = F^* = g(E, h, \nu, a) W^{1/2}$$

with $g_{es} = [8\pi E a^3/(1-\nu^2)]^{1/2}$ and $g_{rc} = \pi a^2 (2\kappa/h)^{1/2}$. In either case, the following are noted: (i) the elastic response is linear because the substrate is assumed to undergo pure bending without film stretching; (ii) both fixed load and fixed grip configurations lead to spontaneous crack growth at $F = F_{\max} = F^*$; (iii) there is no gradual change of contact circle prior to the pull-off event, in that, $c = a$ throughout the loading process. On the contrary, when the thick rigid substrate is replaced by a thin flexible membrane as in our theory, these characteristics will be modified accordingly: (i) the membrane experiences pure stretching without bending so that the elastic response is cubic, a distinct characteristics of thin flexible films [7, 9]; (ii) there exists a stable delamination branch under the fixed grip loading from $w_0 = 0$ and $F = F_{\max}$ to $w_0 = w_0^*$ and $F = F^*$, though fixed load configuration leads to unstable delamination; (iii) the "pull-off" event is also expected here, but only happen when the contact circle contracts to a critical dimension ($c \approx 0.19a$).

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Another interesting consequence of the new model is the predicted existence of a lower energy state (point S in Fig. 2) when the punch is brought into the close proximity of the membrane. From a simple energy consideration, the membrane should jump into contact when the distance from the punch surface falls below w_0^*. The phenomenon is even more pronounced when the punch radius is smaller than $0.19a$. Such a phenomenon was not observed in our simple experiments because of the pressure sensitive adhesive, which did not possess any long-range force to perturb the force field in between the punch and the membrane. It is reminiscent of the JKR theory where a "jump into contact" mechanism was predicted and proved experimentally in various systems [4].

The deviation of the measurements from the theory needs further elaboration. The membrane is hitherto assumed to be *thin and flexible* in the analysis, such that $F \propto w_0^n$ with $n = 3$. It was showed earlier that this cubic constitutive relation is valid only when the flexural rigidity of the membrane, $D = Eh^3/12(1-\nu^2)$, is negligible and when $w_0 \gg h$ [7]. If the ratio (w_0/h) is small at a lower F, bending is more dominant and n reduces gradually from 3 to 1. The membrane used in the present studies fell in the intermediate range and it was shown earlier that $n \approx 2$ [10]. In fact, if the initial loading region of Figure 2 is scrutinized, it is found that the experimental n is roughly 2, rather than the theoretical 3. Another possibility leading to the discrepancies could be the visco-elastic behavior of both the membrane and the adhesive. The fact that the average stress approximation yields a better curve fit than the conical profile model could be due to an overestimation of the elastic energy in the latter.

A simple 1-dimensional equivalence, or an inverted V-peel test [11], can be constructed to compare with the 2-D models above. The loading configuration is similar to Figure 1, though the geometry is no longer axisymmetric but rectangular. A tape of width b is adhered onto a substrate with two fixed edges, $2l$ apart (instead of $2a$ in Fig. 1), and two free edges. The punch has a rectangular flat end wider than the tape. As the punch moves away from the membrane, delamination propagates along the tape length leading to a contact length of $2l_c$ (instead of $2c$ in Fig. 1). Since the membrane is thin and flexible, it supports only stretching and the profile, therefore, has to be linear. The crack front is assumed to be straight rather than the anti-clastic geometry in reality. The constitutive law without delamination and at small debonding angle was found to be cubic again, with

$$F = 2bE'h\left(\frac{w_0}{l - l_c}\right)^3 \tag{11}$$

Upon delamination, the mechanical energy release rate is given by

$$\begin{aligned} G &= \left(\frac{3}{8}\right)\frac{Fw_0}{b(l - l_c)} \\ &= \frac{3}{4}E'h\left(\frac{F}{2E'hb^4}\right)^{4/3} \\ &= \frac{3}{4}E'h\left(\frac{w_0}{l - l_c}\right)^4 \end{aligned} \tag{12}$$

In a fixed load configuration, G is independent of l_c indicating that once F reaches F^* with $F^* = 2.172\ [a(Eh)^{1/4}\ W^{3/4}]$, the crack propagates under a neutral equilibrium until decohesion is completed. In a fixed grip configuration, l_c decreases gradually as the punch moves away from the tape, while the external force is constant throughout at F^*. "Pull-off" occurs when the contact width reduces to a line contact ($l_c = 0$).

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5. CONCLUSION

The elastic solution for a flat punch in adhesive contact with a thin flexible membrane is derived and verified by simple experiments. A "pull-off" event similar to the Johnson-Kendall-Roberts theory is predicted. The new theory is essential in addressing adhesion between solid bodies and thin membranes. Further experiments are being conducted to test the various assumptions of the theory.

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